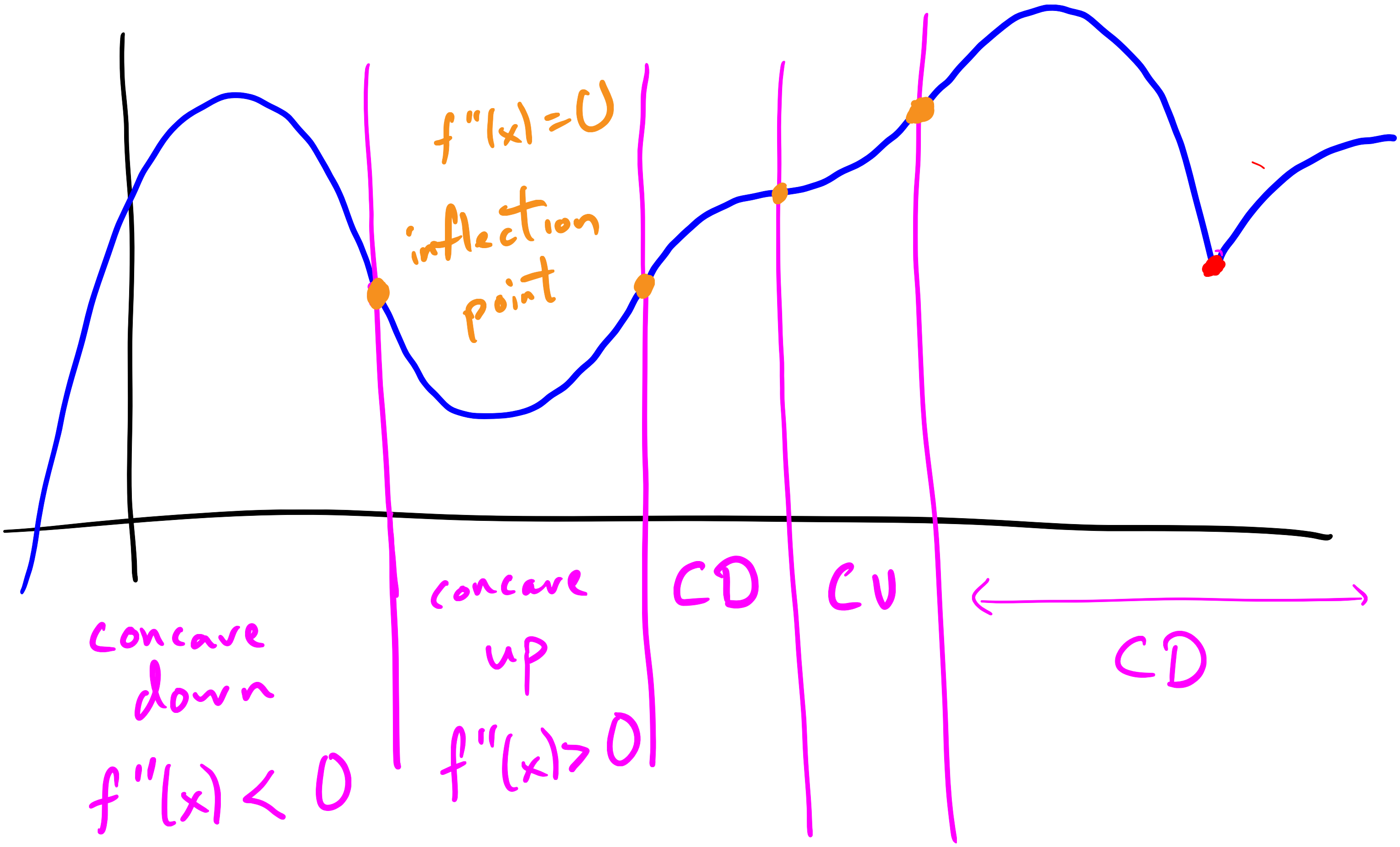
<u>Def</u>: If the graph of f lies above all of its tangent lines on an interval I, we say f is <u>concave up</u> on I. If the graph of f lies below all of its tangent lines on an interval I, then we say f is <u>concave down</u> on I.

Concavity test

1. If $f''(x) > 0$ for all x in I, then f is concave up on I.
2. If $f''(x) < 0$ for all x in I, then f is concave down on I.

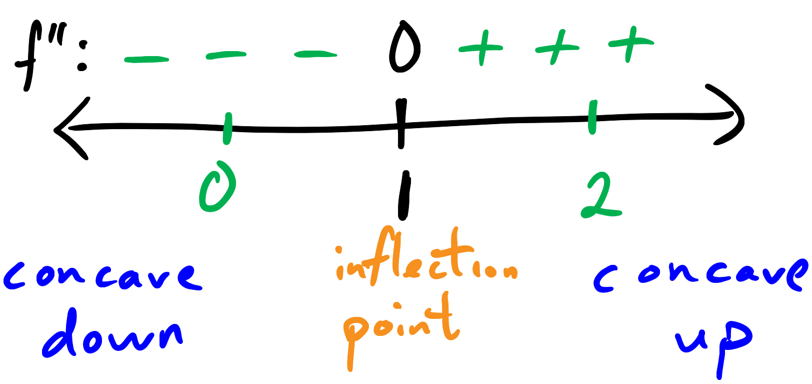
<u>Def</u>: A point P on a curve $y = f(x)$ is called an <u>inflection point</u> if the concavity of f changes at P.



<u>Ex</u>: Find the intervals of concavity and inflection points of $f(x) = x^3 - 3x^2 - 9x + 4$

<u>Sol</u>: $f'(x) = 3x^2 - 6x - 9$

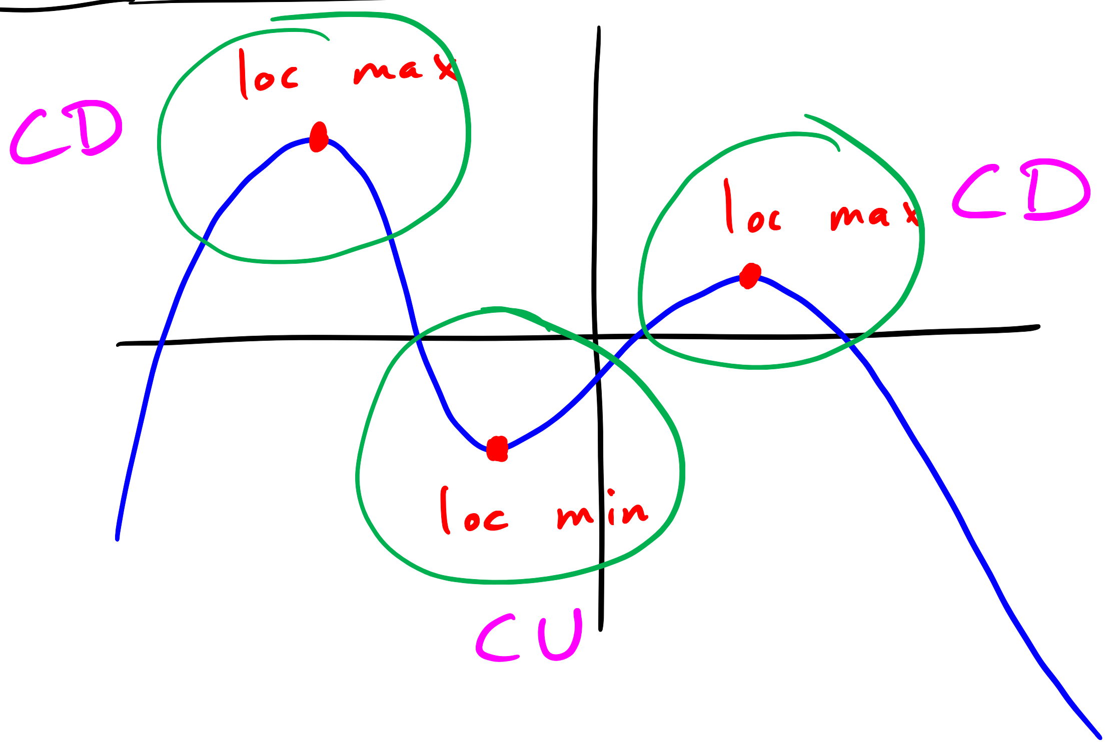
$f''(x) = 6x - 6 = 0 \longrightarrow x = 1$



f is concave down on $(-\infty, 1)$
f is concave up on $(1, \infty)$

$f(1) = 1 - 3 - 9 + 4 = -7$

f has an inflection point at $(1, -7)$



Second Derivative Test

Suppose $f'(c) = 0$ and f'' is continuous near c.

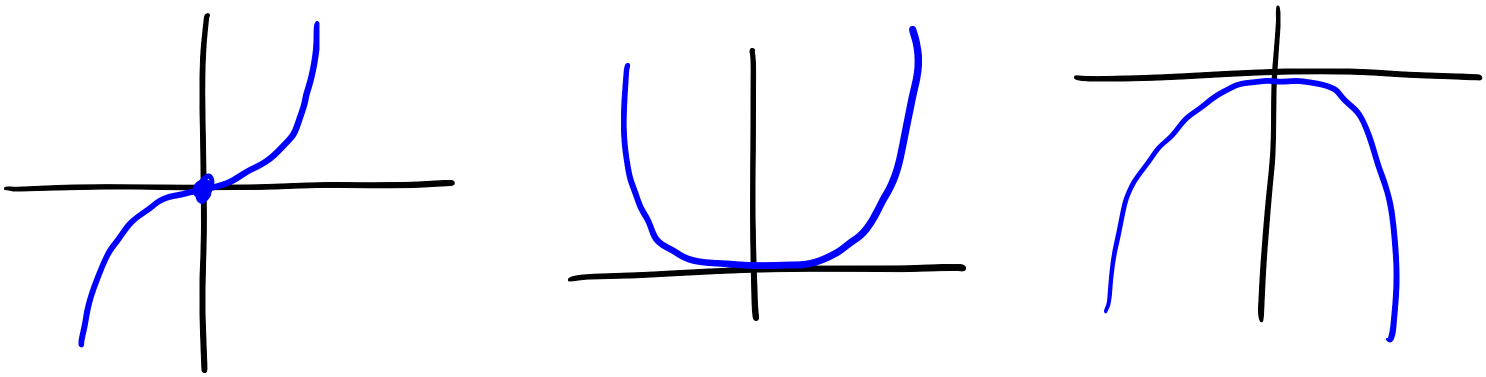
① If $f''(c) > 0$, then c is a local minimum.

② If $f''(c) < 0$, then c is a local maximum.

③ If $f''(c) = 0$, the test fails.

In all 3, $f''(0) = 0$

$f(x) = x^3$ $\qquad$ $f(x) = x^4$ $\qquad$ $f(x) = -x^4$



Ex: Find and classify the local extrema of

$$f(x) = 2x^3 - 9x^2 + 12x - 3$$

Sol:

$$f'(x) = 6x^2 - 18x + 12$$
$$= 6(x^2 - 3x + 2)$$
$$= 6(x-2)(x-1) = 0$$

Crit. #s: $x = 2, 1$

$$f''(x) = 12x - 18$$

Crit. #	$f''(x)$	Class.
2	+	local minimum
1	−	local maximum